**NARAYANA ENGINEERING COLLEGE::NELLORE/GUDUR**

**Academic year: 2020-21**

**Complex Variable & Transforms**

**Year: II B. Tech / Semester: I Branch:ECE,EEE**

**TWO MARKS QUESTIONS WITH ANSWERS**

**UNIT I**

**1. Define analytic function**.

a)Let a function f(z) be derivable at every point z in an neighbourhood of

i.e (z) exists for all z such that < where >0.Then f(z) is said to

be analytic at .

**2.** **Define entire function**.

a)Let D be a domain of complex numbers .If f(z) is analytic at every z D.

f(z) is said to be analytic in the domain D.If f(z) is analytic at every point z

on the complex plane , f(z) is said to be an entire function.

**3. Write properties of Analytic functions**.

a) (i) If f(z) and g(z) are analytic functions, then fg, fg and f/g are also

analytic functions .provided g(z)0 .

(ii) Analytic function of an analytic function is analytic.

(iii) An entire function of an entire function is entire.

(iv) Derivative of an analytic function is itself analytic .

**4. Define Harmonic function.**

a) Solutions of Laplace equations having continuous ,second order partial

derivatives are called Harmonic functions. Their theory is called potential theory. Hence ,the real and imaginary parts of an analytic function are Harmonic functions. Thus the functions satisfying the Laplace equation = 0 are known as Harmonic function.

**5 .Define Conjugate Harmonic function**.

a)If two Harmonic functions u and v satisfy the Cauchy – Riemann equations in a domain D and they are the real and imaginary parts of an analytic function f in D, then v is said to be a Conjugate Harmonic function of u in D.

(OR) Two Harmonic functions u and v which are such that u+iv is an analytic function are called

Conjugate Harmonic function .In other words , if f(z)= u+iv is analytic and if U and v satisfy Laplace’s equation . Then u and v are called Conjugate Harmonic functions.

**6. Write Cauchy’s-Riemann equations in Cartesian form**.

a) The derivative of the function f(z) =w =u(x,y)+v(x,y) to exist for all values of

z in domain R are

1. , are continuous functions of x and y in R.
2. = , = -

The above relations are known as C-R equations.

**7. Write C-R equations in polar form** .

a)If f(z) =f(r ) = u(r,) and f(z) is derivable at =

then = , = -

**8.Show that z2 is analytic for all z**.

a) Let f(z) = u+iv = z 2  =(x+iy) 2 = x2- y2 +i2xy

so that u = x2- y2 , v= 2xy

we have = 2x , 2y

= -2y , =2x

= and = -

The C-R equations are satisfied ⇒f(z) is an analytic function.

9**. Find whether f(z) = sinx siny - i cosxcosy is analytic or not.**

a) Let f(z) = u+iv= sinx siny - i cosxcosy

Then u = sinxsiny v= -cosxcosy

= cosx siny = sinx cosy

= sinx cosy = cosx siny

Thus = and -

i.e C-R equation’s are not satisfied

Hence f(z) is not analytic .

**10.Find where the function w = ceases to be analytic .**

a) Given w =

= = - , if z 1

For z=1, does not exist . so, w is analytic everywhere except at the point

Z=1 which is a singular point of f(z).

**11. Find all values of k, such that f(z) = (cosky +I sinky) is analytic.**

a) Let f(z) = u+iv = ex (cosky+isinky)

Then u(x,y)= cosky v(x,y)= sinky

= cosky = - k sinky

= sinky = k cosky

The C-R equations are satisfied if =

Which is true for k = 1

and = - , which is true for k = 1

Thus f is analytic when k=1

**12. Determine whether the function 2xy+i (x2 – Y2 ) is analytic.**

a) Let f(z)= u+iv=2xy+i (x2 – Y2 )

so u(x,y) = 2xy and v(x,y) = x2 – Y2

=2y , =2x and =2x,= -2y

Hence C-R equations are not satisfied .

f(z) is not analytic.

**13. Define Orthogonal trajectories.**

a)Two cures intersecting at a point p are said to be orthogonal,if their tangents

are perpendicular at p.Two families of curves, or trajectories are orthogonal if

each curve of the first family is orthogonal to each curve of the second family

where even an intersection occurs. Orthogonal families occur in many context .

paralles and meridians on a globe are orthogonal ,as are equipotential and

electric lines of force.

Every analytic function of f(z) =u+iv defines two families of curves

U(x,y)= and v(x,y)= forming an orthogonal system .

**14. Define conformal mapping** .

a) Suppose two curves C, in the z plane intersect at the point p and the corresponding curves and in the w –plane intersect at .I f the angle of intersection of the curves at p is the same as the angle of intersection of the curves at in magnitude and sence , then the transformation is said to be conformal.

**15.Define critical point.**

a)A point at which (z)=0 is called a critical point of the transformation.

**16.Explain about Translation w=z+c.**

a)W=z+c where c is any complex constant is a translation

suppose z=x+iy , c= +i and w= u+iv

Then, we get u+iv=(x+iy)+(+i )

Comparing real & imaginary parts.

U=x+ and v= y+

i.e if p(x,y) is a point in the z-plane then (x+,y+)is the corresponding

point in the w- plane .Thus , if the w-plane is superimposed on z-plane ,the

figure is shifted through a distance which is given by c.Thus ,this transformation

maps a figure in the z plane to a figure of same shape and size in w-plane. In

particular circles are mapped on to circles under this transformation.

**17.Find analytic function whose imaginary point is v(x,y)=2xy**

a) 2y

=2x

(z)= + i =2x + i 2y

Put x=z, y=0

(z) = 2z

Integrating ,we get f(z) = z2+c.

**18.Find the fixed points of the transformation is w=**

a) put w= z ,we have z=

on solving , the fixed points are z= -1,+i,-i.

**19.The harmonic conjugate of u(x,y)=x2- y2 is v=?**

a) Using Milne –Thompson method , (z) = 2z

integrating, f(z) = z2+c.

=x2- y2 +i (2xy +k)

Conjugate harmonic , v = 2xy +k.

**20. Find the fixed points of the transformation w =**

(a) The fixed points are given by putting w=z

We get z2-i2 =0 i.e z = i ,-i

The fixed points are z = i ,-i

**UNIT-II**

**1. Define zero of  order.**

Ans:- If an analytic function f(z) can be expressed in the form f(z)=(z-a)m  (z)

where (z) is analytic and (a) 0,then z=a is called zero of m th order

of the function f(z). simple zero is a zero of order one.

**Eg:-**  f(z) =(z-1)3 then z=1 is a zero of order 3 of f(z).

**2. State Cauchy’s integral theorem**.

Ans:- Let f(z) =u(x,y)+iv(x,y)be analytic on and within a simple closed contour c and

let (z) be continuous there. Then .

**3. State Cauchy’s integral formula.**

Ans:- Let f(z) be an analytic function everywhere on and with in a closed contour c.

If z=a is any point with in c,then f(a)= dz

Where the integral is taken in the +ve sense around c.

**4. State the generalization of Cauchy’s integral formula.**

Ans:- If f(z) is analytic on and with in a simple closed curve c and if a is any point

with in c then

(a) = dz

**5. Evaluate )dz along the real axis from z=0 to z=1**

Ans:- Let z= x+iy then dz =dx+idy

Along OA, y=0 z=x and dz=dx

Also x varies from 0 to 1

)dz = )dz= dx

=

**6. State the Taylor’s theorem.**

Ans:- Let f(z) be analytic at all points with in a circle with centre at a and radius r.

Then at each point z with in

f(z)= f(a)+f1(a)(z-a)+ (z-a)2 +……….+ (z-a)n +…… -------🡪(1)

i.e the series on the right hand side in (1)convergence to f(z) whenever |z-a|<

which is known as Taylor’s series expansion of f(z) about z=a.

**7. State the Laurent’s theorem.**

Ans:- Let be two circles given by |z 1 – a|= and |z1 –a|= respectively

where and z1 is any point on .

Let f(z) be analytic on and throughout the region between the two

Circles . Let z be any point in the ring shaped region between the circles.

Then f(z)= + ------🡪(1)

Where = dz1 -----------🡪(2)

= dz1  --------🡪(3)

Where the integrals are taken around in the anti -clock wise sence.

**8. If c is any simple closed curve, evaluate .**

Ans:- The function , is analytic everywhere and hence in particular on and within any simple closed curve c. Hence, by Cauchy’s theorem



**9. Define isolated singularity with an example.**

Ans:- A point z=a is called an Isolated singularity of an analytic function f(z),if

(a)f(z) is not analytic at the point z=a

(b)f(z) is analytic in the deleted nbd of z=a,i.e there exists a nbd of the point

Z=a which contains no other singularity.

**Eg:-** If f(z)= , then z=i are two Isolated singular points of f(z).

**10. If c is any simple closed curve, evaluate .**

Ans:- The function , is analytic everywhere and hence in particular on and within any simple closed curve c. Hence, by Cauchy’s theorem



**11. dz =**

Ans:- Consider the problem is, evaluate dz along .

Along , 

dz=

**12. Define pole of a complex function with example.**

Ans:- If there exists a positive integer *n* such that  then is called a pole of order n.

**Eg:-** The function  {\displaystyle f(z)={\frac {3}{z}}}has a pole of order 1 or simple pole at *{\displaystyle z=0}z=*0.

**13. Evaluate  along .**

Ans:- Along , 

Also *x* varies from *0* to *1*

= 

**14. Define residue.**

Ans:- The coefficient of in the Larent’s series expansion of f(z) about the isolated

singularity z=a is called the residue of f(z) at that point . Thus the residue of f(z)

at z=a is .From Larent’s series , we know that the coefficient is given by

=

i.e = 2=2[Residue of f(z) at z=a ]= 2

**15. State residue theorem.**

Ans:- If is analytic within and on a closed curve *c*, except at a finite number of poles  within *c* and  be the residues of at these poles, then



(or)

(sum of the residues at the poles within *c*).

**16. Determine the poles of the function z/cosz**

Ans:- The poles of f(z) =z/cosz are given by cos z=0

i.e z=(2n+1)/2 ,n being zero or an integer

i.e z=(2n+1)/2 , n=0,……….

Hence these are simple poles of f(z).

**17. Define residue at a pole of order m.**

Ans:- If f(z) is analytic with in a curve c and has a pole of order m at z= is

[]

**18. [z2 /(z-1)(z+2)2]=**

Ans:- 



**19. [z2 /(z-1)2(z+2)]=**

Ans:- 



**20. State Cauchy’s residue theorem.**  Ans:- If is analytic within and on a closed curve *c*, except at a finite number of poles  within *c* and  be the residues of at these poles, then



or (sum of the residues at the poles within *c*).

**UNIT-III**

**1.Write the conditions for existence of Laplace transform of a function. [DEC 2016]**

a)We finding the Laplace transform of elementary functions. It can be noticed that the

integral exists under certain conditions, such as s>0 or s>a etc . In general ,the function

f(t) must satisfying the following conditions for the existence of the lapiace transform .

(i)The function f(t) must be piece – wise conditions or sectionally continuous in any

Limited interval 

(ii)The function f(t) is of exponential order .

**2.Define unit impulse function. [NOV 2017,DEC 2016]**

a) The Dirac Delta Function can be defined as follows:



The limit of as is denoted by and is called Dirac Delta Function.

Thus , the unit impulse function is defined as follows:



Such that 

Laplace Transform of Dirac Delta Function is 

**3. Find  [JUNE 2017, JUNE 2016]**

a) Given **** =  (by first shifting theorem)



**4.Find  [JUNE 2016]**

a) Given **** = (by Laplace transform of division by t)



**5.Find **

a) Given ****  = 



**6.Find **

a)Given ****

Let 







Hence 

**7.Find L{sinhbt}**

a)Given L{sinhbt}

by shifting property L{ f(t)} =

here f(t)=sinhbt , = ; a=a

L{sinhbt} = =

**8.State change of scale property for Inverse Laplace transform .**

a)If L{f(t)}= (s),then { (as)} = f(t/a),a>0.

**9.Find L{coshbt}**

a) Given L{coshbt}

by shifting property L{ f(t)} =

here f(t)=coshbt , = ; a=a

L{coshbt} = =

**10.Find {}**

a)Given {}

by second shifting theorem { (s)}=

(or)

{ (s)}= f(t-a) H(t-a)

here (s) = 1/, f(t)=t f(t-3)=t-3 ;a=3

so, {} =

**11.Find L{t}**

a)Given L{t} = L{} = L{1+cos2t} = [ L{1}+ L{cos2t} ] = [ +]

**12.Find {}**

a)Given {} = { }

by second shifting theorem { (s)}=

(or)

{ (s)}= f(t-a) H(t-a)

here (s) = ; f(t) = ; a=3

{} = (or) = H(t-a)

**13.Find inverse transform of  [NOV 2017]**

a) 

**14.Find L{sin2tcost}**

a)Given L{sin2tcost} = L{2sin2tcost}= L{sin3t+sint} = [L{sin3t}+L{sint}]

= []

**15.State first shifting theorem for Laplace transform.**

a)If L{f(t)}= (s) then L{f(t)}= (s-a) ,s-a>0

L{f(t)} = (s+a),s+a>0

which is called first shifting theorem for laplace transform.

**16.Find L{cos2t}**

a)Given L{ cos2t}

by shifting property L{ f(t)} = (s+a)

here f(t)= cos2t ; (s) = ;a=1

L{ cos2t} = =

**17.State the unit step function. [JUNE 2017]**

a)The unit step function is defined as H(t-a) (or) U(t-a)=

which is known as the unit step function at t=a

Then Laplace transform of unit step function is

i.e L{H(t-a)}=

**18.State the second shifting theorem for Laplace transform.**

a)If L{f(t)}= (s) ; and g(t) =

then L{g(t)} = (s)

which is known as second shifting theorem.

**19.Define Convolution theorem.**

a)If L{f(t)}= (s) and L{g(t)}= (s) then L{f(t)\*g(t)} = (s) (s)

(or) { (s) (s)}= f(t)\*g(t) =

which is called Convolution theorem.

**20.State the Laplace transform of derivatives.**

a) If f(t) is continuous and of exponential order and (t) is sectionally continuous then

the Laplace transform of (t) is given by L{(t)}=s (s) – f(0) where (s) = L{f(t)}

**21. Find L{}**

a) Given L{}

by first shifting theorem If L{f(t)}= (s) then L{f(t)}= (s-a)

here f(t)= a=1 ; (s) =

then L{} = = =

**22.Find L{}**

a) Given L{} = } ds= ds

=

= = log()

**23.Find L{}**

a) Given L{} = L{ } = = = =

**24. Find }**

a)Given } = {} = (by first shifting theorem)

**25.Find {}**

a) Given {} = {}

= =

**26.Find {}**

a) Given {} = {}

= (by shifting property)

**UNIT IV**

**1. Define Dirichlet conditions for expansion of f(x) in Fourier series.**

a) A function f(x) has valid Fourier series expansion of the form

 ,where  are constants, provided:

(i) f(x) is periodic, single valued and finite.

(ii) f(x) has a finite number of discontinuities in any one period.

(iii) f(x) has at the most a finite number of maxima and minima.

**2.Write the parseval’s formula for fourier series.**

a) The Fourier series for the function  converges uniformly in the interval  is given by

****

**3.Express f(x) =x as a fourier series from **

a)Given f(x) =x which is odd function in 

Hence in its fourier series expansion ,the cosine terms are absent and only sine terms

are present .



From equation (1) gives , 

**4. What is formula for Half range Cosine series. Or what is fourier even function in (-π,π). (June 2017)**

a) . Where  ,

**5.Find sine series of f(x) =k in (0,**

a)Given f(x)=k

The fourier sine series expansion is 



Hence 

**6. If f(x)=  then find .**

a)Given **f(x)= **





**7. State whether y= tan x can be expanded as a Fourier series . If so how ? If not why ?**

a) tan x cannot be expanded as a Fourier series .Since tan x not satisfies Dirichlet’s

conditions.(tan x has infinite number of infinite discontinuous).

**8. What is formula for Half range Sine series.**

a)  . where 

**9. Findif  in .**

a) 

 

( Since is even function).

**10. Findif  in .**

a)



(Since Cosx is –ve in π/2<x<π )

**11. Find constant bn for x Sinx in [-π, π].**

a) Given function f(x)= xSinx is even function.

Thus the Fourier series of f(x) contains cosine terms only.Hence bn=0.

**12.Find a0, if f(x) = ex in **

a) 

**13. Find the constant a0 of the Fourier series for the function f(x) = k , .**

a) 

**14. If f(x) = |x| expanded as a Fourier series in  then Find a0.**

a) 

Since |x| is even function.

**15.What is the formula for fourier cosine series and fourier sine series.**

a)The forier cosine series is f(x)= 

The fourier sine series is f(x)= 

**16.If f(x) =x in (0,2 find the fourier coefficient .**

a)The fourier coefficient 

**17.If f(x)=  then find the fourier coefficient .**

a) The fourier coefficient = 

**18.What is the formula for the half range cosine and sine series for the function f(x) in (0,l).**

a) The half range cosine series for the function f(x) in (0,l) is

f(X)= 

where  ,  dx

The half range sine series for the function f(x) in (0,l) is

f(x)=  where 

**19.Define periodic function .**

a) A function f(x) is said to be of period T or to be periodic with period T>0 if for all x

f(x+T) = f(x) and T is the least of such values.

**20. Findif  in** .

a) 

 

( Since is even function).

**21. If f(x) = in (-1,1) then find the fourier coefficient of bn . (June 2017)**

**a).** Since f(x) is an even function in (-1,1), =0.

**UNIT-V**

**1.Write the Complex form of Fourier integral.**

a)Fourier integral in complex form :

f(x) = f(t) dtdp

**2.Write any two properties of Fourier transform.**

a) (i) Linearity property:

If F(p) and G(p) are fourier transforms of f(x) and g(x) respectively , then

F{ a f(x) + b g(x) } = a F(p) + b G(p) Where a &b are constants

(ii)Change of scale property :

If F(s) is the complex Fourier transform of f(x) then 

(iii)Shifting property :

If F(s) is the complex Fourier transform of f(x) then 

**3. What is **

a)  = 

= =  = 

And  =

=

**4. Write the formula for the inverse fourier transform of F(s) in **

a) The inverse fourier transform of F(p) is given by

f(x) = dp

**5. Write the formula of the fourier Cosine integral of f(x).**

a) Fourier Cosine integral is 

**6. Define finite fourier Sine and Cosine transforms and their inversion formula in 0<x<L.**  (June 2017)

a) Finite Fourier Sine transform is 

Inverse Fourier Sine transform is 

Finite Fourier Cosine transform is 

Inverse Fourier Cosine transform is 

**7. State Fourier integral theorem.**

a) Fourier integral theorem states that 

**8. Write Fourier sine and cosine integral of f(x).**

a) Fourier sine integral is 

Fourier Cosine integral is 

**9. Write infinite Fourier Transform of f(x).**

a) 

**10. State the change of scale property.**

a) If F(s) is the complex Fourier transform of f(x) then 

**11. State the shifting property.**

a) If F(s) is the complex Fourier transform of f(x) then 

**12. State Modulation theorem.**

a) If F(s) is the complex Fourier transform of f(x) then 

**13. Find Fourier Cosine transform of.**

a) Given ****

We know that 

**14. Find Fourier Sine transform of.**

**a)** Given ****

We know that 

**15. Write Sine and Cosine transform of f(x).**

a) 



**16.Write linearity property of fourier transform.**

a)If F(p) and G(p) are fourier transforms of f(x) and g(x) respectively .

then F{a f(x)+b g(x)} = a F(p)+b G(p) where a&b are constants .

**17. What is the formula for parseval’s identity for fourier transforms .**

a) If F(p) and G(p) are fourier transforms of f(x) and g(x) respectively , then

(i) = 

(ii)= 

**18.State the convolution theorem for fourier transform .**

a)If F{f(x)} and F{g(x)} are the fourier transform of f(x) and g(x) respectively .then the fourier

transform of the convolution of f(x) and g(x) if the product of their fourier transforms

F{f(x)\*g(x)}=F{f(x)} F{g(x)}

**19.Write the Dirichlet ‘s conditions for fourier transform .**

a) A function f(x) is said to satisfy Dirichlet’s conditions in the interval (a,b), if

(i) f(x) defined and is single valued function except possibly at a finite number of points

in the interval (a,b) and

(ii) f(x) and are piece wise continuous in (a,b)

**20.Find the fourier sine transform of f(x)=**

a)The fourier sine transform of f(x) is given by 



Hence 